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THE THEORY OF FORCED GYROSCOPIC HORIZONS

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The Principle of Forced Gyroscopic Stabilization

In designing gyroscope supports and general construction, one employs two different principles to take off readings from gyroscopic apparatus and stabilizers. The first of these principles consists of the maximum "load-relieving" of the gyroscopic sensitive element from the disturbing forces which may have caused the element's declination from the position to be maintained. To do this, one employs various, often rather intricate anti-frictional devices and the readings are taken off from the minimum mechanical load of the sensitive element. Both problems are often solved with the help of a tracking system, which reproduces all the sensitive element's movements relative to the ship or airplane in which the apparatus is set up. One of the difficulties here consists in the fact that the errors in the sensitive element are accumulated along with the errors in the tracking system and the high stability of the gyroscopic system cannot be fully utilized.

The other principle may be called the principle of forced stabilizers and consists of employing a "load-relieving" servomotor or electromagnets which contribute forces that compensate for the frictional forces in the supports and the slow reaction of the apparatus in transmitting the readings. In order to control or regulate automatically the "load-relieving" of the apparatus, small relative movements are utilized in the system of the gyroscopic sensitive element, which movements being caused by the

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disturbing forces. During these movements, moreover, gyroscopic moments arise which react against the disturbing force and govern the stabilization at the first moment so long as the "load-relieving" apparatus is not operating. In this case there is no need, either for the tracking system or for a super delicate designing of the external supports of the sensitive element, and the readings can be taken immediately with it. Here the frame of the gyroscopic sensitive element assumes at the same time the role of the tracking system.

An example of a forced stabilizer is the well known gyroscopic "nodometer", the description of which is in E. A Sperry's book /17/. The book also shows that with the aid of two such Sperry stabilizers it is possible to obtain an artificial horizon; that is, a platform able to be maintained in a position close to the horizontal.

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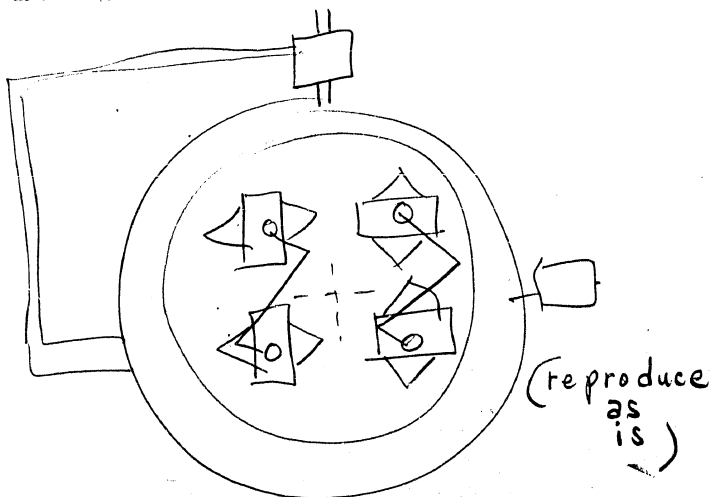
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The Equations of Motion of a Forced Gyroscopic Horizon

One of the possible schemes is presented in the flat diagrammatic sketch. The η axis of the Cardan wheel is directed along the longitudinal axis of the ship. The stabilizable platform can be rotated relative to the wheel around the second Cardan x axis. The angles of rotation around these axes are denoted by alpha α and beta β and are assumed to be small. S_x and S_y are the "load-relieving" motors. The $\xi\eta$ plane of the true coordinate system $\xi\eta\zeta$ which is connected with the ship, is assumed to be horizontal. The xy plane of the second coordinate system xyz is the plane of the platform. The housings of the four gyroscopes of natural moments H_x, H'_x, H_y, H'_y, can rotate relative to the platform around the perpendiculars to their axes, so that the axes of rotation of the gyroscope remain in the xy plane. Since the housings are joined in pairs as antiparallelograms, then for small angles of precession σ, σ' of the first two gyroscopes we have approximately the relation $\sigma = -\sigma'$. Analogously for the other two gyroscopes we have $\tau = -\tau'$. The arrows H_x and H_y in the diagram represent also the slide runners of the potentiometers, which control the "load-relieving" motors. The moments, transferable by the motors to the Cardan axes, are $S_x \tau = S_y \sigma$, where S_x and S_y are considered constant. In order to create the regulating forces, the horizontal pendulums K_x and K_y are employed, which regulate the motors E_x and E_y placed on the precessional axes of the gyroscopes. During any movement of the ship of constant velocity v with respect to the ship course, making a constant angle ψ with the direction of the north, the inclinations of the pendulums are proportional to the projections $p \alpha$ and $p \beta$ of the weight p of the pendulum bob upon the axes x and y. If the speed is changed and the

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ship's trajectory has a radius of curvature ρ , then the pendulums are acted upon by other forces; namely, the forces of inertia $\rho v^2/g\rho$, $-\rho v/g$. Therefore the moments generated by the regulating motors are $K_x (\alpha + \frac{\omega y}{g})$ and $K_y (\beta - \frac{y}{g})$, where K_x and K_y are assumed to be constant and $\omega = \frac{v}{\rho}$ is the angular velocity of circulation (ship's turning on the arc of radius R).

As for the equations of motion, it is possible to connect the equations of moments relative to the x, y, axes for the whole system and the equations of moments relative to the whole precession for the gyroscopic pairs H_x, H'_x, H_y, H'_y:

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Here A, B are the moments of inertia of the whole system relative to the x, y axes and the inertial terms of form $C_1 \ddot{\sigma}$, $C_2 \ddot{\tau}$ in the two latter equations are disregarded, in as much as the corresponding masses are not large: Γ_x, Γ_y, Γ_1, Γ_2 are the gyroscopic moments, whose expressions may be obtained from the formulas (2.10) in the article by Ya. N. Roytenberg [2]. Designating by U the angular velocity of the Earth's rotation, by R the Earth's radius, by ϕ the latitude of the place; taking into consideration the various distinctions in the remaining designations; and assuming that $H_x = H'_x$, $H_y = H'_y$ — then we shall find that:

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Substituting in the equations of motion and disregarding the products of $U \sin\phi$ by $\alpha, \beta, \sigma, \tau$, we then obtain:

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(1)

(2)

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$$(1)$$

$p\ 291$

$$(2)$$

Hence it is seen that, if a ship moves with constant velocity v
with respect to a constant course (v = const, ψ = const, $\dot{v}$ = ω = 0),
then the coordinates $\alpha, \beta, 0, 0$ vary aperiodically and tend to a
constant limiting value $\alpha*$, $\beta*$, 0, 0 where:

$$(3)\quad p\ 291$$

These quantities are velocity deviations; they are small if K_x,
K_y are sufficiently large. For $\dot{v} \neq 0$ and $\omega \neq 0$ the apparatus
receives several ballistic deviations which can be studied by the usual
methods $\boxed{3}$.

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The Possibility of Obtaining a 84-Minute Period and of Compensating for the Ballistic Deviations

We shall modify the preceding scheme by uniting the pendulum K_x with the gyroscopic H_y, H'_y and the pendulum K_y with the gyroscopic H_x, H'_x. Equations (1) are not modified, but in equations (2) the expressions $K_x(\alpha + \frac{\omega}{g})$ and $K_y(\beta - \frac{v}{g})$ will vary slightly in places where we perform "connections" in such a manner that the second expression has the minus sign. Assuming that:

$$\frac{K_y}{2H_x} = \underline{\qquad\qquad}$$

(4) p 292

we obtain instead of (2):

(5) p 292

or setting:

(6) p 292

multiplying by 1 and i and then adding:

(7) p 292

We are then led to a new unknown:

(8) p 292

where the quantity:

(9) p 292

is the velocity deviation; we note that $\dot{\psi} = -\omega$ and we write:

$$v = \sqrt{\frac{g}{R}} = 1.241 \cdot 10^{-3} \, \text{sec}^{-1};$$

(10) p 292

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hence we obtain the following transformation equation:

(11) $p292$

If we select the parameters such that $k = \pm \nu$, then
the equation becomes homogeneous; this indicates that the apparatus
in this case does not possess ballistic deviations. Thus, for
the given system compensation is possible according to M. Shuler,
since the system acts during compensation as a gyroscopic pendulum
with a period of precession:

$$T = \frac{2\pi}{\nu} = 84.4 \; min.$$

(12) $p292$

It does not present any difficulties to provide for dampening
of the free procession. In order to do this it is necessary
to combine both of the discussed variants of the system by joining
each of the pendulums with both gyroscopic pairs and by selecting
in a suitable manner the transmission ratios of signals.

LITERATURE

/1./ E. Sperry. Applied Gyrodynamics. New York. pp 125-128. 1933.

/2./ Ya. N. Roytenburg. Poly-gyroscopic Vertical. Applied Math.
& Mech. X No. 1, 1946.

/3./ B. V., Bulgakov. Applied Theory of Gyroscopes. M-L. 1939.

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